459. Proposed by C. N. SCHMALL, New York City.

In a right triangle ABC, right-angled at C, a point F is taken in the side CB and perpendiculars CD and FE are dropped on the hypothenuse AB. Prove $AD \cdot AE + CD \cdot EF = \overline{AC}^2$.

CALCULUS.

When this issue was made up, solutions had been received for numbers 366–377

378. Proposed by ELBERT H. CLARKE, Purdue University.

The area of the curved surface generated by the revolution about OX of the portion of the curve $y = x^n$ which extends from the origin to the point (1, 1) is given by the formula

$$A = 2\pi \int_0^1 x^n \sqrt{1 + n^2 x^{2n-2}}\, dx.$$

Our geometric intuition would tell us that the limit of this area as n becomes infinite is π. Give a strict analytic proof that

$$\operatorname*{Lim}_{n=\infty} \int_0^1 x^n \sqrt{1 + n^2 x^{2n-2}}\, dx = \tfrac{1}{2}.$$

379. Proposed by C. N. SCHMALL, New York City.

Express the equation of the folium, $x^3 + y^3 = 3axy$, in parametric form and find the area of the loop.

(From E. B. Wilson's *Advanced Calculus*, p. 296, ex. 5.)

MECHANICS.

When this issue was made up, solutions had been received for numbers 297, 301, and 302

303. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

A pile-driver weighing 500 pounds falls through 10 ft. and drives a pile weighing 400 pounds 3 inches into the ground. Show that the average force of the blow is $11{,}111\frac{1}{9}$ pounds.

NUMBER THEORY.

When this issue was made up, solutions had been received for numbers 224, 225, 226, and 229

228. Proposed by HERMON C. KATANIK, Indianapolis, Ind.

Deduce a formula for the difference between any two squares, and thus show that (1) The difference between any two consecutive squares is of the form $2p + 1$; (2) The difference between any two squares is even or odd according to whether they are separated by an odd or even number of squares; (3) The differences of the squares of the consecutive terms of any arithmetic progression form another arithmetic progression.

229. Proposed by WALTER C. EELLS, U. S. Naval Academy.

If p and q are integers and p is prime and positive, find the condition on q that the equation $p^x = qx$ shall have integral solutions, solve for x, and show that for a special value of p it has two solutions for a given q, otherwise only one.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

418. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Form the algebraic equation whose roots are

$$a_1 = 2\cos\left(\frac{2\pi}{15}\right), \quad a_2 = 2\cos\left(\frac{4\pi}{15}\right), \quad a_3 = 2\cos\left(\frac{8\pi}{15}\right), \quad a_4 = 2\cos\left(\frac{14\pi}{15}\right).$$